Wilsonian RG and Redundant Operators in Nonrelativistic Effective Field Theory

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In a Wilsonian renormalization group (RG) analysis, redundant operators, which may be eliminated by using field redefinitions, emerge naturally. It is therefore important to include them. We consider a nonrelativistic effective theory (the so-called "pionless" Nuclear Effective Field Theory) as a concrete example and show that the off-shell amplitudes cannot be renormalized if the redundant operators are not included. The relation between the theories with and without such redundant operators is established in the low-energy expansion. We perform a Wilsonian RG analysis for the *off-shell* scattering amplitude in the theory with the redundant operator.

I. INTRODUCTION

Effective field theories[1, 2, 3, 4, 5, 6] are useful in exploring the low-energy physics with only relevant degrees of freedom and operators. Chiral Perturbation Theory (χPT)[7] is a prominent example, in which only meson degrees of freedom are treated. The idea of χPT has been applied to the sectors with baryons. The Nuclear Effective Field Theory (EFT)[8, 9] is that with two baryons and more. The Nuclear EFT is a promising "model-independent" approach to nuclear physics, based on general principles of quantum field theory. Because of its inherent nonperturbative nature, however, we are still unable to understand the basic structure of the theory, though some preliminary successes have been reported. See Refs.[10, 11] for reviews.

One of the most important issues is how to determine the power counting. There are mainly two distinct power counting schemes: the one due to Weinberg[8, 9] is based on the naive dimensional analysis for the construction of the "effective potential" which is used in the Lippmann-Schwinger equation, and the other proposed by Kaplan, Savage, and Wise[12, 13] takes into account that the scattering length is unnaturally long and is implemented by the so-called *power divergence subtraction*. It has been shown that Weinberg's power counting is inconsistent[14] while the KSW power counting fails to converge in certain channels[15]. The present status may be summarized as that Nuclear EFT works "pretty well if one follows a patchwork of power counting rules[16]." Clearly, a *principle* of systematizing the power counting, which tells the correct one *before* doing numerical calculations, is needed. (See Ref.[17] for a possible solution.)

We think that such a principle may be provided by a Wilsonian renormalization group (RG) approach[18, 19]. In a Wilsonian approach, one does not need to prescribe a particular power counting. We hope that the RG flow itself determines how to treat the operators.

A Wilsonian RG approach has been performed by Birse and collaborators[20, 21]. They consider the RG equation for a *energy-* (as well as momentum-) *dependent* potential. (See Ref.[22] for a similar but distinct approach and Ref.[23] for the comparison.) Note that the notion of Wilsonian RG should be independent of the on-shell nature. Note also that the potential approach has the so-called off-shell ambiguities, which may be a problem when one considers the three-nucleon systems. A completely field theoretical treatment is desired. We have been developing it, and will be reported elsewhere[24]. In this paper, we will give some preparatory remarks that are indispensable for understanding the development.

In a completely field theoretical treatment, a Wilsonian RG transformation generates all kinds of operators consistent with the symmetry. Among them, there are operators which are proportional to the (tree-level) equations of motion. These operators, which we call "redundant," are usually eliminated by a field redefinition. In order for the RG equations not to contain the couplings for such redundant operators, the field redefinition should follow the RG transformation.

It is well-known that the physical S-matrix is independent of the choice of the field variables[25]. One might therefore think that the field redefinition mentioned above is a trivial procedure. In this paper, we illustrate that it is far from trivial. We consider a simple nonrelativistic system (known as "pionless" EFT) as a concrete example and show how the redundant operator is eliminated without affecting physics. (Note that we do not claim that pionless EFT is particularly interesting. As easily seen, our results are very general and the simplest example illustrates the essential points.) We show that the coupling constants are transformed nontrivially. With this effect taken into account, we obtain the RG equations for the reduced set of coupling constants.

In Sec. II, we review the standard argument for the independence of the physical S-matrix on the choice of the field variables, and emphasize that there are contributions from the measure. In Sec. III we give an explicit calculation of the two nucleon scattering amplitude in

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the pionless Nuclear EFT, and show that the off-shell amplitude for the theory without the redundant operator cannot be renormalized. We further show that it is possible to eliminate redundant operators without modifying physics, but the coefficients of the other operators must be modified in a nontrivial way. In Sec. IV we derive the RG equations and renormalize the *off-shell* scattering amplitude for the theory with the redundant operator. In Sec. V we summarize the results.

II. INDEPENDENCE OF THE PHYSICAL S-MATRIX ON THE CHOICE OF THE FIELD VARIABLES

It is well known that the physical S-matrix elements are independent of the choice of the field variables[25]. A standard argument goes as follows[26, 27, 28]. Consider a scalar field theory for simplicity. We consider the following point transformation,

$$\phi \to \Phi \equiv f(\phi), \tag{2.1}$$

and the two n-point functions,

$$\begin{aligned} G_{(n\phi)}(x_1,\cdots,x_n) &= \langle T\phi(x_1)\cdots\phi(x_n)\rangle, &(2.2)\\ G_{(n\Phi)}(x_1,\cdots,x_n) &= \langle T\Phi(x_1)\cdots\Phi(x_n)\rangle \\ &= \langle Tf(\phi)(x_1)\cdots f(\phi)(x_n)\rangle. &(2.3)\end{aligned}$$

According to the LSZ formalism, the S-matrix elements in the Φ-theory are obtained by multiplying the Klein-Gordon operator $\Box + m^2$ (where m is the mass of the particle) to each leg of the connected part of $G_{n\Phi}$ and considering the on-shell limit $p^2 \to m^2$. Because only the single particle propagation has the pole, this procedure picks up the Φ-ϕ transition part of all the possible diagrams, giving rise to the same S-matrix elements. (A possible wave function renormalization factor is canceled anyway in obtaining the S-matrix elements.)

The above argument itself is of course valid but it might cause some confusion in practice. The point is best illustrated in the path integral formulation. The n-point function $G_{(n\phi)}$ may be given by

$$G_{(n\phi)}(x_1,\cdots,x_n) = \int \mathcal{D}\phi\phi(x_1)\cdots\phi(x_n)e^{i\int d^4x\mathcal{L}(\phi)}, \tag{2.4}$$

while $G_{(n\Phi)}$ by

$$\begin{aligned} &G_{(n\Phi)}(x_1,\cdots,x_n) \\ &= \int \mathcal{D}\phi f(\phi)(x_1)\cdots f(\phi)(x_n)e^{i\int d^4x\mathcal{L}(\phi)}, \end{aligned} \tag{2.5}$$

not by

$$\int \mathcal{D}\Phi\Phi(x_1)\cdots\Phi(x_n)e^{i\int d^4x\tilde{\mathcal{L}}(\Phi)}, \tag{2.6}$$

where

$$\tilde{\mathcal{L}}(f(\phi)) = \mathcal{L}(\phi). \tag{2.7}$$

The difference comes from the Jacobian factor, $\det|\mathcal{D}f(\phi)/\mathcal{D}\phi|^{-1}$, which introduces new interactions in the Φ-theory.

In a fermionic theory, ψ and $\psi^\dagger$ are essentially a canonically conjugate pair. A field transformation which mixes ψ and $\psi^\dagger$ is not a point transformation nor (generally) a canonical transformation, and thus leads to a nontrivial Jacobian factor. As we will see in the following section, the field redefinition which eliminates redundant operators is of this kind.

In many cases, the Jacobian may be disregarded[29, 30]. In perturbation theory it gives higher order contributions, and when dimensional regularization is employed it does not contribute at all. In any case, the Jacobian may be represented as a series of local interactions[34], which are already present in EFT, so that it is absorbed in the definitions of the coupling constants. The nonrelativistic theory that we will consider in the next section, however, allows us to calculate the amplitudes nonperturbatively, and in order to perform a Wilsonian RG analysis, dimensional regularization should not be employed. An explicit calculation reveals the nontrivial character of the contributions from the measure.

III. PIONLESS NUCLEAR EFT AS A CONCRETE EXAMPLE

We consider the following simple nonrelativistic effective field theory,

$$\begin{aligned} \mathcal{L} &= N^\dagger\left(i\partial_t + \frac{\nabla^2}{2M}\right)N - C_0\left(N^TN\right)^\dagger\left(N^TN\right) \\ &\quad + C_2\left[\left(N^TN\right)^\dagger\left(N^T\overleftrightarrow{\nabla}^2N\right) + h.c.\right] + \cdots \end{aligned} \tag{3.1}$$

where N is a “spinless nucleon” field[13],

$$\overleftrightarrow{\nabla}^2 = \overrightarrow{\nabla}\cdot\overrightarrow{\nabla} - 2\overleftarrow{\nabla}\cdot\overrightarrow{\nabla} + \overleftarrow{\nabla}\cdot\overleftarrow{\nabla}, \tag{3.2}$$

and ellipsis denotes higher order operators. Though the spin is neglected in this theory, the results are identical to those for the spin singlet channel. Note that, because of the nonrelativistic nature, the production of anti-particles is suppressed and the particle number is conserved. In particular, the six-nucleon operators, such as $\left(N^\dagger N\right)^3$ do not contribute in the two particle sector. Note also that the theory is invariant under Galilean boost. We have to keep this symmetry.

The Lippmann-Schwinger (LS) equation for the two particle scattering amplitude $\mathcal{A}(p^0,\boldsymbol{p}_1,\boldsymbol{p}_2)$ in the center-of-mass frame is given by

$$\begin{aligned} &-i\mathcal{A}(p^0,\boldsymbol{p}_1,\boldsymbol{p}_2) = -iV(\boldsymbol{p}_1,\boldsymbol{p}_2) \\ &+\int\frac{d^3k}{(2\pi)^3}\left(-iV(\boldsymbol{k},\boldsymbol{p}_2)\right)\frac{i}{p^0-\boldsymbol{k}^2/M+i\epsilon}\left(-i\mathcal{A}(p^0,\boldsymbol{p}_1,\boldsymbol{k})\right), \end{aligned} \tag{3.3}$$

where V is the vertex in momentum space,

$$V(\boldsymbol{p}_1, \boldsymbol{p}_2) = C_0 + 4C_2\left(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2\right) + \cdots, \qquad (3.4)$$

and p^0 is the center-of-mass energy of the system, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the relative momenta in the initial and final states respectively. We consider the off-shell amplitude for the moment so that p^0, $\boldsymbol{p}_1$, and $\boldsymbol{p}_2$ are unrelated.

By introducing a cutoff Λ on the relative momentum $\boldsymbol{k}$ and expanding the amplitude up to including quadratic terms in each momentum,

$$\mathcal{A}(p^0, \boldsymbol{p}_1, \boldsymbol{p}_2) = x(p^0) + y(p^0)(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2) + z(p^0)\boldsymbol{p}_1^2\boldsymbol{p}_2^2, \quad (3.5)$$

we may solve the LS equation in a closed form[31, 32]:

$$\begin{aligned} x &= \left(C_0 + 16C_2^2 I_2\right)/D, & (3.6)\\ y &= 4C_2\left(1 - 4C_2 I_1\right)/D, & (3.7)\\ z &= 16C_2^2 I_0/D, & (3.8) \end{aligned}$$

with

$$D = 1 - C_0 I_0 - 8C_2 I_1 + 16C_2^2 I_1^2 - 16C_2^2 I_0 I_2, \qquad (3.9)$$

where I_n are integrals defined by

$$I_n = -\frac{M}{2\pi^2}\int_0^\Lambda dk \frac{k^{2n+2}}{k^2 + \mu^2}, \quad \mu = \sqrt{-Mp^0 - i\epsilon}. \quad (3.10)$$

At low energies, we can expand the inverse of the *on-shell* amplitude in powers of the momentum, and fit to the scattering length α and the effective range r,

$$\mathcal{A}^{-1}\big|_{on-shell} = -\frac{M}{4\pi}\left[-\frac{1}{\alpha} + \frac{1}{2}rp^2 + \mathcal{O}(p^4) - ip\right], \quad \text{with } p = \sqrt{Mp^0} = |\boldsymbol{p}_1| = |\boldsymbol{p}_2| \quad (3.11)$$

to determine the couping constants C_0 and C_2 as functions of the cutoff Λ. It is however impossible to do so *off-shell*, because μ is an independent variable. We have three equations for the two coupling constants. In other words, the conventional method does not renormalize the off-shell amplitude.

The notion of renormalization should be independent of whether we renormalize the amplitude on-shell or off-shell. It is clear that the troublesome terms contain the factor $\boldsymbol{p}_1^2 + \mu^2$ or $\boldsymbol{p}_2^2 + \mu^2$, i.e., $\nabla^2 + Mp^0$ in coordinate space. As we will show shortly, the operator proportional to the "equation of motion" is necessary to renormalize the theory off-shell.

In reality, the operators such as

$$2B\left[\left(N^T N\right)^\dagger N^T\left(i\partial_t + \frac{\nabla^2}{2M}\right)N + h.c.\right], \qquad (3.12)$$

(where B is a coupling constant) must be included in the EFT Lagrangian, because they are local operators satisfying the symmetry of the theory. But these operators are usually discarded by using the field redefinition,

$$N \to N - 2BN^\dagger\left(N^T N\right). \qquad (3.13)$$

As emphasized in Sec. II, this field redefinition gives rise to an extra factor coming from the measure, which should be treated carefully in our case.

It would be possible to calculate the contributions from the measure explicitly if we properly regularize the products of the operators in the field redefinition and the measure itself. Here, we instead follow an indirect way.

Because the transformation is local and does not contain derivatives, the extra interaction coming from the measure must be represented as (an infinite set of) local operators. Furthermore, such operators should satisfy the symmetry of the theory. However, all the possible interactions are already included in the EFT Lagrangian! The net effect is therefore to change the coupling constants. We can determine the changes of the coupling constants by demanding that the transformed theory (without the B-interaction) have the same physical amplitude as that of the original theory (with the B-interaction).

Let us consider the ("original") theory with the B-interaction (3.12) included. The LS equation may be solved in the similar way, with the vertex V being replaced by

$$\begin{aligned} V'(p^0, \boldsymbol{p}_1, \boldsymbol{p}_2) &= C_0' + 4C_2'\left(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2\right) \\ &\quad - 2B\left(p^0 - \left(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2\right)/2M\right) + \cdots. \end{aligned} \qquad (3.14)$$

The amplitude

$$\mathcal{A}'(p^0, \boldsymbol{p}_1, \boldsymbol{p}_2) = x'(p^0) + y'(p^0)(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2) + z'(p^0)\boldsymbol{p}_1^2\boldsymbol{p}_2^2, \qquad (3.15)$$

may be easily obtained by noting that V' is obtained by substituting $C_0 \to C_0' - 2Bp^0$ and $4C_2 \to 4C_2' + B/M$ into V, resulting

$$\begin{aligned} x' &= \frac{1}{D'}\left((C_0' - 2Bp^0) + \left(4C_2' + \frac{B}{M}\right)^2 I_2\right), & (3.16)\\ y' &= \frac{1}{D'}\left(4C_2' + \frac{B}{M}\right)\left(1 - \left(4C_2' + \frac{B}{M}\right)I_1\right), & (3.17)\\ z' &= \frac{1}{D'}\left(4C_2' + \frac{B}{M}\right)^2 I_0, & (3.18) \end{aligned}$$

with

$$\begin{aligned} D' &= 1 - \left(C_0' - 2Bp^0\right)I_0 - 2\left(4C_2' + B/M\right)I_1 \\ &\quad + \left(4C_2' + B/M\right)^2 I_1^2 - \left(4C_2' + B/M\right)^2 I_0 I_2. \end{aligned} \qquad (3.19)$$

We now demand that the denominator $D'(C_0', C_2', B)$ be proportional to $D(C_0, C_2)$ *off-shell*,

$$D(C_0, C_2) = R(B)D'(C_0', C_2', B). \qquad (3.20)$$

We require the off-shell proportionality because the relation between (C_0, C_2) and (C_0', C_2', B) should be independent of the on-shell nature, and the normalization of the amplitude may be affected by field transformations.

The requirement has the solution,

$$C_0 = R\left(C_0' + \left(4C_2' + \frac{B}{M}\right)^2 L_5\right) - \frac{L_5}{L_3^2}\left[1 - \sqrt{R}\left(1 - \left(4C_2' + \frac{B}{M}\right)L_3\right)\right]^2, \quad (3.21a)$$

$$C_2 = \frac{1}{4L_3}\left[1 - \sqrt{R}\left(1 - \left(4C_2' + \frac{B}{M}\right)L_3\right)\right], \quad (3.21b)$$

$$R = \left[1 - 2\frac{B}{M}L_3\right]^{-1}, \quad (3.21c)$$

where

$$L_n = -\frac{1}{n}\frac{M\Lambda^n}{2\pi^2}. \quad (3.22)$$

It is easy to show that the *on-shell* amplitudes are actually identical for both theories if the above condition is satisfied.

Some remarks are in order. Firstly, by setting $B = 0$, the solution reduces to the trivial one; $C_0 = C_0'$, $C_2 = C_2'$, and $R = 1$. Secondly, C_0 and C_2 are linear in C_0' and C_2'. Thirdly, C_0 contains C_2'. Finally, the coefficient of C_0' in C_0 and that of C_2' in C_2 are not equal. The former two are what one would expect, while the latter two are difficult to understand. One would naively expect

$$C_0 = A(B)C_0' + \delta_0(B), \quad C_2 = A(B)C_2' + \delta_2(B), \quad (3.23)$$

because the changes of the coupling constants (δ_0 and δ_2) come form the Jacobian for the field redefinition (3.13), it should contain only the coupling constant B. (The factor $A(B)$ comes from the "wave function renormalization" due to the Jacobian contribution proportional to the kinetic term.) We suspect that this dependence of the Jacobian on the coupling constants C_0' and C_2' comes from the definition of the composite operator $N^\dagger\left(N^T N\right)$.

IV. RG EQUATIONS

Once we obtain the complete two particle scattering amplitude, it is easy to derive the RG equations for C_0', C_2', and B, by demanding (the inverse of) the amplitude is, when expanded in $\boldsymbol{p}_1^2$, $\boldsymbol{p}_2^2$, and μ^2, independent of the cutoff Λ.

Let us introduce the dimensionless coupling constants γ_0, γ_2, and β as,

$$\gamma_0 = \frac{M\Lambda}{2\pi^2}C_0', \quad \gamma_2 = \frac{M\Lambda^3}{2\pi^2}4C_2', \quad \beta = \frac{\Lambda^3}{2\pi^2}B. \quad (4.1)$$

We obtain the following RG equations,

$$\Lambda\frac{dX}{d\Lambda} = -(1-X)(Y+3X^2)/X, \quad (4.2a)$$

$$\Lambda\frac{dY}{d\Lambda} = Y(6X^3 - 5X^2 + 2XY - Y)/X^2, \quad (4.2b)$$

$$\Lambda\frac{dZ}{d\Lambda} = Y^2/X^2 - 3Z + 6XZ + 2YZ/X, \quad (4.2c)$$

where $X = 1+(\gamma_2+\beta)/3$, $Y = \gamma_0-(\gamma_2+\beta)^2/5$, and $Z = 2\gamma_2 + (\gamma_2+\beta)^2/3$. This set of equations has a nontrivial fixed point $(X^*, Y^*, Z^*) = (1, -1, -1)$ corresponding to

$$(\gamma_0^*, \gamma_2^*, \beta^*) = \left(-1, -\frac{1}{2}, \frac{1}{2}\right), \quad (4.3)$$

beside the trivial one, $(X, Y, Z) = (1, 0, 0)$ corresponding to $(\gamma_0, \gamma_2, \beta) = (0, 0, 0)$. At the nontrivial fixed point, the theory is of course scale invariant[33] and the scattering length is infinite. In the real world, we are a bit away from the fixed point.

We may linearize the RG equations around the fixed point by substituting

$$\gamma_0 = \gamma_0^* + \epsilon_0, \quad \gamma_2 = \gamma_2^* + \epsilon_2, \quad \beta = \beta^* + \epsilon_\beta, \quad (4.4)$$

so that we have

$$\Lambda\frac{d\epsilon_0}{d\Lambda} = -\epsilon_0 - 2\epsilon_2 - 2\epsilon_\beta, \quad (4.5a)$$

$$\Lambda\frac{d\epsilon_2}{d\Lambda} = -2\epsilon_0 - \frac{2}{3}\epsilon_2 - \frac{5}{3}\epsilon_\beta, \quad (4.5b)$$

$$\Lambda\frac{d\epsilon_\beta}{d\Lambda} = 2\epsilon_0 + \frac{8}{3}\epsilon_2 + \frac{11}{3}\epsilon_\beta. \quad (4.5c)$$

These equations are easily solved. We have found the following eigenvalues and the corresponding eigenvectors;

$$2 : \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}, \quad 1 : \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad -1 : \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}. \quad (4.6)$$

It is important to note that there is a negative eigenvalue -1; namely, there is a relevant combination of the operators.

By using them, we have

$$\epsilon_0 = 2a\left(\frac{\Lambda}{\Lambda_0}\right)^2 + c\left(\frac{\Lambda_0}{\Lambda}\right), \quad (4.7a)$$

$$\epsilon_2 = a\left(\frac{\Lambda}{\Lambda_0}\right)^2 - b\left(\frac{\Lambda}{\Lambda_0}\right) + c\left(\frac{\Lambda_0}{\Lambda}\right), \quad (4.7b)$$

$$\epsilon_\beta = -4a\left(\frac{\Lambda}{\Lambda_0}\right)^2 + b\left(\frac{\Lambda}{\Lambda_0}\right) - c\left(\frac{\Lambda_0}{\Lambda}\right), \quad (4.7c)$$

where a, b, and c are infinitesimal dimensionless constants and Λ_0 is a parameter of mass dimension. Inserting these solutions, we obtain the (renormalized) off-shell amplitude near the nontrivial fixed point,

$$\mathcal{A}'^{-1}(p^0, \boldsymbol{p}_1, \boldsymbol{p}_2)\Big|_* = -\frac{M\Lambda_0}{2\pi^2}c - \frac{Mb}{\pi^2\Lambda_0}\mu^2 - \frac{M\mu}{4\pi} + \cdots, \quad (4.8)$$

where ellipsis denotes higher orders in a, b, and c. It is important to note that the pole is independent of the relative momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ of the center-of-mass system, as it should be. This point is not clear from the (renormalized) on-shell amplitude.

Finally we describe the method of obtaining the RG equations for the reduced set of coupling constants. Start with the set of the coupling constants $(C_0, C_2) \equiv (C_0', C_2', B = 0)$, and infinitesimally lower the cutoff Λ by $\delta\Lambda$. It transforms $(C_0', C_2', B = 0)$ to $(C_0' + \delta C_0', C_2' + \delta C_2', \delta B)$ according to the RG equations (4.2). Now we invoke the equivalence relation (3.21) to eliminate δB;

$$(C_0' + \delta C_0', C_2' + \delta C_2', \delta B) \to (C_0 + \delta C_0, C_2 + \delta C_2). \quad (4.9)$$

The resulting RG equations are identical to those obtained from the Λ-independence of the *on-shell* amplitude, as expected.

V. CONCLUSION

In this paper, we showed that the elimination of redundant operators generically has the Jacobian contribution and that it plays an important role in the Wilsonian RG analysis. The "pionless" EFT was considered as a concrete example. It was shown that the *off-shell* amplitudes cannot be renormalized without the redundant operator (the "B-interaction"). Off-shell renormalization is important, particularly in analyzing the three-body systems.

We established the equivalence relation between the theories with and without the redundant operator. The Jacobian contribution was shown to have very peculiar features.

We performed a Wilsonian RG analysis based on the two particle scattering amplitude. It is important to note that a Wilsonian RG analysis is impossible without the redundant operators, because the RG transformations generate all possible operators which satisfy the symmetry of the theory, including the redundant operators.

We derive the equivalence relation by directly comparing the (nonperturbative) amplitudes, but in many cases it is impossible to obtain them. A direct method of calculating the Jacobian factor is desired.

The physical significance of the Wilsonian RG analysis of Nuclear EFT will be reported elsewhere[24]. Here we just mention that the RG flow determines the *phase structure* of the nuclear force and the inverse of the scattering length may be identified with the *order parameter*; it characterizes the relevant direction near the nontrivial fixed point.

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